Key: Yellow highlight = required component

Simulation Using Repeat and Randomize

Subject Area(s)
Computer Science

Associated Unit

Lesson Title **Simulation Using Repeat and Randomize**

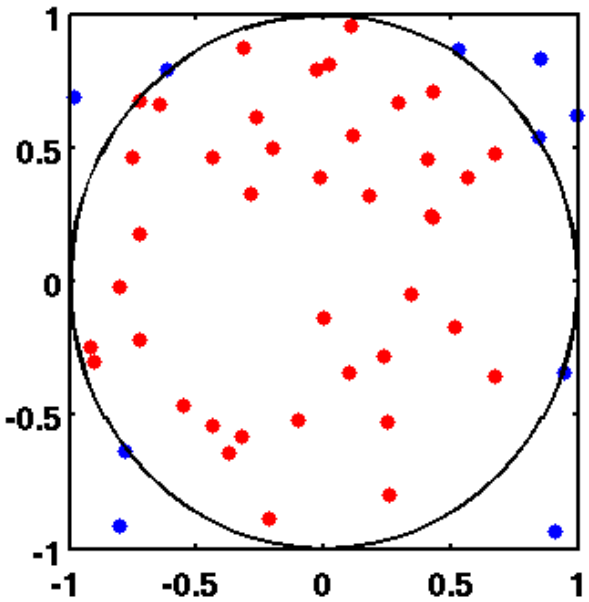

Image 1 Monte Carlo Square and Circle

Image file:
http://upload.wikimedia.org/wikipedia/commons/thumb/b/b0/MonteCarloIntegrationCircle.png/220px-MonteCarloIntegrationCircle.png

ADA Description: An illustration of a Monte-Carlo simulation that plots random dots on a circle inscribed in a square. The square sides are divided by units that follow the coordinate system with origin at the center of the figure.

Source/Rights: "MonteCarloIntegrationCircle" by Yoderj - My own fair hand and Matlab. Licensed under Public domain via Wikimedia Commons - http://commons.wikimedia.org/wiki/File:MonteCarloIntegrationCircle.png#mediaviewer/File:MonteCarloIntegrationCircle.png

Caption: How to approximate the value of PI?

Lesson # N/A

Lesson Dependency NONE

Time Required 90 minutes

Group Size 2 minimum / 4 maximum / one computer per group

Expendable Cost per Group US $0

Summary

This lesson, planned for second quarter immediately after the chapter for programming iterations, allow students to use their knowledge of Java control statements and the Math class to familiarize themselves with a Monte Carlo based simulation algorithm.

A Monte Carlo Simulation is a way of approximating the value of a function where calculating the actual value is difficult or impossible. It uses random sampling to define constraints on the value and then makes a sort of "best guess."[2]

Students will write a simulation program that uses a simple loop and Java random's function to approximate the value of PI graphically instead of geometrically.

Engineering Connection

A simulation program uses the computer to simulate an activity in the real world (or an imaginary one). Simulations are commonly used for predicting climate change, analyzing traffic, picking stocks, and many other applications in science and business. In many simulations, one or more loops are used to modify the state of a system and observe the changes [e.g.: Gauss-Seidel] while the Random class of the Java library implements a random number generator, which produces numbers that appear to be completely random.[1]

Monte Carlo methods (or Monte Carlo experiments) are a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results; typically one runs simulations many times over in order to obtain the distribution of an unknown probabilistic entity. The name comes from the resemblance of the technique to the act of playing and recording results in a real gambling casino. They are often used in physical and mathematical problems and are most useful when it is difficult or impossible to obtain a closed-form expression, or infeasible to apply a deterministic algorithm. Monte Carlo methods are mainly used in three distinct problem classes: optimization, numerical integration and generation of draws from a probability distribution.[3]

Engineering Category =

Choose the category that best describes this lesson's amount/depth of engineering content:

1. Relating science and/or math concept(s) to engineering

Keywords

Iterative, loops, estimation, geometry, Monte Carlo, ratio, pi, programming, random, simulation

Educational Standards (List 2-4)

State STEM Standard (required)

> FLDOE Science, Technology, Engineering & Math (STEM) Curriculum Frameworks - Secondary Courses/Programs, 2014-15, 63.03/63.08/73.04/73.06/73.07, Information Technology STEM Programs - Game/Simulation/Animation Programming (8208300)
>
> FLDOE Science, Technology, Engineering & Math (STEM) Curriculum Frameworks - Degree & Certificate Programs, 2014-15, 04.0X, Information Technology STEM Programs - Computer Programmer (CCC - 0511020200)

ITEEA Standard (required)

International Technology and Engineering Educators Association: Technology, 2000, 17/D, The Designed World

CCSS Standard (strongly recommended)
Mathematics Florida Standards 2014
Domain: STATISTICS & PROBABILITY: MAKING INFERENCES & JUSTIFYING CONCLUSIONS

Cluster 2: Make inferences and justify conclusions from sample surveys, experiments, and observational studies
MAFS.912.S-IC.2.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
Cognitive Complexity: Level 2: Basic Application of Skills & Concepts

Cluster 2: Make inferences and justify conclusions from sample surveys, experiments, and observational studies
MAFS.912.S-IC.2.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
Cognitive Complexity: Level 2: Basic Application of Skills & Concepts

Learning Objectives

After this lesson, students should be able to:

- To program loops with either the while or for statements
- To avoid infinite loops and off-by-one errors
- To use common loop algorithms
- To use algorithms to represent the code for a simulation
- To apply loops and random values to implement simulations of real events

Materials List

Each group needs:

- Integrated rapid development tool (e.g. BlueJ)
- One computer with JDK installed per group

To share with the entire class:

- Powerpoint presentation
- Modified Monte Carlo Simulation Worksheet (read activity below)
- Assessments

Introduction / Motivation (5E – Engage)

In the fight against Global Warming, scientists and engineers have to deal with CO2 which is the main greenhouse gas emitted from the combustion of carbon-based fossil fuels in automobiles and power plants. One of humanity's pressing needs is to capture and sequester CO2 to contain serious global environment problems. One smart cost-effective way to try new materials that will capture and sequester CO2 is using math and computer simulations to try those materials before investing in their manufacturing. Go to http://youtu.be/_ph3ceMm75Q to see a simulation of a MOM in action.

One set of methods used to create simulations is called Monte Carlo (named after the famous Monaco casino.) As the name aptly suggests, the methods are based on probability and statistics. The simulation presented above uses one of those methods. However, other Monte Carlo methods allow for calculations that otherwise might be very difficult to determine. For example, how can we measure the size of a solar flare? Monte Carlo to the rescue.

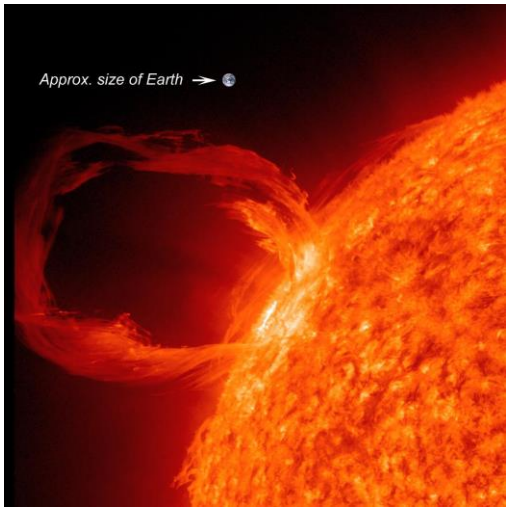

Figure 2

Image file: http://www.nasa.gov/images/content/525037main_FAQ1-orig_full.jpg

ADA Description: A solar eruptive prominence as seen in extreme UV light on March 30, 2010 with Earth superimposed for a sense of scale.

Source/Rights: Copyright © NASA/SDO

Caption: Figure 2. Solar Flare

In this lesson students will use their knowledge of Java control statements and the Math class to familiarize themselves with a Monte Carlo algorithms that will show how to calculate the value of PI graphically instead of geometrically, and subsequently they could use it to explain how to approximate the size of the solar flare.

Lesson Background & Concepts for Teachers (5E – Explain)

Students should be comfortable with the following:

- Computing the area of a square and a circle
- Using the equation of a circle: x^2 + y^2 = r^2
- Representing the circle using the coordinate system
- Using Java's decision statements
- Writing Java's loop (repetitive) statements
- Using Java's Math library and/or the Random class

Vocabulary / Definitions

Word	Definition[4]
Iterative	involving repetition
loops	a series of instructions (as for a computer) that is repeated until a terminating condition is reached
random	chosen, done, etc., without a particular plan or pattern
simulation	something that is made to look, feel, or behave like something else especially so that it can be studied or used to train people
ratio	the relationship that exists between the size, number, or amount of two things and that is often represented by two numbers

Associated Activities (5E – Explore)

Activity materials for this lesson are adapted from an existing activity in the teachengineering.org site. The name (and link) for the activity is, A Chance at Monte Carlo take note that the recommended material might require modifications.

List of Recommended material:

Monte Carlo Presentation (Ignore the statistical sections)

Monte Carlo Simulation Worksheet (Ignore the statistical sections)

Instructions

A simple Monte Carlo Simulation can be used to calculate the value for π. If you had a circle and a square where the length of a side of the square was the same as the diameter of the circle, the ratio of the area of the circle to the area of the square would be π/4. So, if you put this circle inside the square and select many random points inside the square, the number of points inside the circle divided by the number of points inside the square and the circle would be approximately π/4. Write [on separate paper an algorithm that will generate the simulation. Use the algorithm to write] a function to run a simulation like this with a variable number of random points to select. Also, show the results of a few different sample sizes. [2]

Outcome for this activity minus algorithm (solutions may vary)

Java

```
public class MC {
	public static void main(String[] args) {
		System.out.println(getPi(10000));
		System.out.println(getPi(100000));
		System.out.println(getPi(1000000));
		System.out.println(getPi(10000000));
		System.out.println(getPi(100000000));
	}
	public static double getPi(int numThrows){
		int inCircle= 0;
		for(int i= 0;i < numThrows;i++){
			//a square with a side of length 2 centered at 0 has
			//x and y range of -1 to 1
			double randX= (Math.random() * 2) - 1;//range -1 to 1
			double randY= (Math.random() * 2) - 1;//range -1 to 1
			//distance from (0,0) = sqrt((x-0)^2+(y-0)^2)
			double dist= Math.sqrt(randX * randX + randY * randY);
			//^ or in Java 1.5+: double dist= Math.hypot(randX, randY);
			if(dist < 1){//circle with diameter of 2 has radius of 1
				inCircle++;
			}
		}
		return 4.0 * inCircle / numThrows;
	}
}
```

Output:

```
3.1396
3.14256
3.141516
3.1418692
3.14168604
```

(source: Wikipedia http://rosettacode.org/wiki/Monte_Carlo_methods)

Assessment (5E – Evaluate)

Pre-Lesson Assessment with tentative key
Descriptive Title: Walking Through Memory Lane

How do you find the area of a circle and a square?

PI*(r^2) or PI*(d^2)/4

What are loop statements?

(Answers will vary) One example: They are instructions that will repeat one or more statements (sometimes none) until a terminating condition ends the process.

What is a random value?

(Answers will vary) One example: For numbers, a random value is a real number that has an equal chance to be chosen (i.e without a pattern or reason) from the infinite set of numbers.

How Java generates a pseudo-random value?

It could use the Math random method or the Random class

Using the coordinate system, how do you find the distance from the origin (0,0) to any point?

Sqrt(x2^2 + y2^2)

What is a ratio?

(Answers will vary) One example: Is a non-dimensional (no units) relation usually between two objects' similar properties that is usually represented in fractional form indicating the number of times one property is contained within the other.

How could you estimate the area of any shape inside a square?

(Answers will vary) One example: Using known geometric shapes to overlap the original shape)

Post-Introduction (Formative) Assessment (Answers will vary)
Descriptive Title: What RU Doing?

Which kind of loop works better for the simulation?

Since we know the number of times the loop repeats, the For loop is a better choice than the While loop which runs until a condition is met.

Why can you write your program to use quarter of a circle instead of the whole circle?

The area of the circle and the area of the square are reduced by 4, so the ratio PI/4 stays the same

What happens if you increase the number of sample points?

The value of PI gets more precise

Why do we use a loop?

Because we need to repeat statements for each point randomly chosen by the program.

Lesson Summary (Summative) Assessment (Answers will vary)
Descriptive Title: That's All Folks

How a while loop works when compared to the For loop?

The While loop will end when a condition is met unlike the For loop that will run for a pre-determined number of times.

How can you simulate the throwing of a die?

You could use the Random class to get a objectName.nextInt(6)+1

Explain how you can approximate the area size of a solar flare using this Monte Carlo method.

View the Monte Carlo presentation for the answer.

Lesson Extension Activities (5E – Extension)

1.) Given the precise value of PI by Java, modify the program to display the approximation error for each varying number of points sample.

2.) The following link ***STP Monte Carlo Estimation*** provides a GUI version to find the area of a quarter circle using Monte Carlo. After downloading the jar file and experimenting with the interface, try to create a similar graphical output for this activity using Java's graphics libraries.

3.) Converging to the square root of a number – This activity shows students how to use Newton's iterative method to find the square root of a number (The algorithm for this method is located here, and the Java code is located at the Rosetta code org website)

References

1.) Horstmann, Cay S. *Java Concepts: Compatible with Java 5, 6, & 7*. Hoboken, NJ: Wiley, 2010. Print.

2.) "Monte Carlo Methods." *Monte Carlo Methods*. Rosettacode.org - GNU Free Documentation License, 16 July 2014. Web. 27 July 2014.

3.) "Monte Carlo Method." *Wikipedia*. Wikimedia Foundation, 26 July 2014. Web. 27 July 2014.

4.) "Dictionary." *Merriam-Webster*. Merriam-Webster, n.d. Web. 25 July 2014.

5.) "Solar Storm and Space Weather - Frequently Asked Questions." *NASA*. NASA, n.d. Web. 28 July 2014.

6.) "A Chance at Monte Carlo." *Home - Www.TeachEngineering.org*. Applying Mechatronics to Promote Science (AMPS) GK-12 Program, Polytechnic Institute of New York University, n.d. Web. 28 July 2014.

Attachments

Follow the links in the lesson plan.

Contributors

Pedro Villavicencio

Space Research group; Department of Chemistry; University of South Florida

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Classroom Testing Information

TBD